Artificial Intelligence—Qual (Fall 2017)

Please read instructions carefully before starting to write, work carefully, and circle your final answers (unless it is obvious what the final answer is). Do not worry if you cannot finish everything. Do not write down disorganized answers in the hope of getting partial credit; it is better to do fewer questions completely right. You can use extra pages. There are 100 points in total. Calculators are not allowed. Good luck!

—Vince
1 Search (25 points total).

In theme park AI-land, the only allowed mode of transportation is to use centrally coordinated self-driving cars. Because of this, there is no need for traffic lights at intersections: the centralized system will determine when each car will cross, in a safe and efficient way. There are still traffic lanes.

Suppose the situation at an intersection is as in Figure 1a. The letters in boxes represent cars waiting to cross in the way indicated by their lane. (We assume there is always at most one car in each lane.) The system works by identifying, in each period, the set of cars that will cross. For the sake of safety, the paths of the cars chosen in a single period may not intersect (moving into the same lane is also intersecting). The goal is to let all the cars cross in the minimum number of periods. No other cars will arrive and nothing will go wrong.

For example, one possibility is to first let D and H cross; then A and E; then C and F; then B and G. This solution requires 4 periods, and so has a cost of 4. (Can you find a solution of cost 3?)

We will solve this problem using search. First we need a useful heuristic. Consider the graph $G$ in which the remaining vehicles are the vertices $V$, and there is an edge between two vehicles if their paths intersect. Figure 1b shows the graph for the instance given above (at the root node of the search tree).
a (10 points). Based on this graph $G$, which of the following is an admissible heuristic for the search problem? Briefly explain your answer.

(i) The size of the largest clique (a clique is a set of vertices such that there is an edge between every two vertices in the set).

(ii) $|E|/2$ (the number of edges in the graph, divided by 2).

(iii) $|V| - \text{mincut}$, where mincut is the minimum number of edges that need to be removed to make the graph disconnected (i.e., so that some two vertices are no longer reachable from each other using the remaining edges).

b (15 points). Solve the instance in Figure 1a using the $A^*$ algorithm, using the admissible heuristic from part a, so that you will find an optimal solution. You should show the entire search tree, and at each node show the value of $g$ (cost incurred so far) and $h$ (the admissible heuristic). You should break ties so that the tree stays as small as possible. The label of a node should be the set of cars that have already crossed, listed alphabetically. E.g., after the first two steps in the 4-step solution above, the state is ADEH. You may assume that in each round a maximal set of cars crosses, i.e., a set to which nothing can be added. E.g., you do not need to generate a node for only letting car D cross in the first round, because it would be possible to add car E in the same round. But letting D and E cross together is maximal, so you should have a node for that. The root node has 7 children (but it will very quickly get better after that if you proceed the right way).
2 First-Order Logic (25 points total).

a (10 points). Translate the following into first-order logic. It will make the next part easier if you use disjunctions (\( \vee \), which means “or”) as much as possible, so that you can use resolution easily. Please use the following relations: \( \text{Wet}(\cdot), \text{Raining}, \text{HasUmbrella}(\cdot), \text{Reckless}(\cdot), \text{SlipsOn}(\cdot, \cdot) \).

1. Alice is not wet.

2. If it is raining, every thing that does not have an umbrella is wet.

3. Every thing that is reckless does not have an umbrella.

4. If the ground is wet, then it is raining.

5. If the ground is not wet, then no thing slips on it.

6. Every thing that slips on the ground is reckless.
b (15 points). \textbf{Prove}, in a sequence of formal steps, that Alice does not slip on the ground. Do this by assuming

(7) Alice slips on the ground.

and deriving a contradiction. Each step should indicate which known propositions you use (at most 2 at a time) and give a new proposition that is unconditionally true (given 1-7 above). For full credit, use \textbf{resolution} wherever possible. (It is possible to use all the axioms in the order given above, each time using the current axiom to update your most recent proposition, via resolution.)
3 Bayesian Networks (25 points total).

Dr. Bayz is investigating a population of mice on a remote island. He finds that some are losing their fur, and others have problems with their teeth. The two features are correlated.

a (3 points). For the above scenario, draw a Bayesian network (without conditional probability tables) with two variables, one called Fur Condition ($F$), and one called Teeth Condition ($T$).

b (3 points). Suppose each of the two variables can take one of four values: Good, Fair, Poor, and Terrible. How many probabilities must be specified, in total, for the conditional probability tables? (Note: if there were only a single variable taking one of $n$ values, then the answer would be $n - 1$, because the value of the final probability could be inferred.)

After painstaking research, Dr. Bayz finds out that some mice have a particular infection, and others do not. This infection appears to cause both problems for the mice, and in fact completely explains the correlation between hair loss and tooth decay in the mice. That is, conditional on Infection ($I$), Fur Condition ($F$) and Teeth Condition ($T$) are independent.

c (3 points). Draw a Bayesian network (without conditional probability tables) with three variables, $I$, $F$, and $T$, that captures the above conditional independence assumption.

d (3 points). How many probabilities must be specified for the conditional probability tables? ($I$ is a binary variable taking value either 1 (infected) or 0 (not infected).)

e (3 points). Suppose we wish to increase the number of values for each of the two health conditions ($F$ and $T$) from 4 to $n$ (but we still keep only two values for $I$). Asymptotically (in big-O notation), how many probabilities need to be specified for the first network? How many for the second?
f (10 points). Suppose we know (only) the following probabilities:

\[ P(F = \text{Good}|I = 1) = 0.5, \ P(T = \text{Good}|I = 1) = 0.4, \ P(F = \text{Good}|I = 0) = 0.8, \ P(T = \text{Good}|I = 0) = 0.9. \]

For each of the following, calculate the probability, or state that it is impossible to know given the limited information given.

\[ P(F = \text{Good}) \]

\[ P((F = \text{Good}) \text{ and } (T = \text{Good})|I = 1) \]

\[ P(I = 1|(F = \text{Good}) \text{ and } (T = \text{Good})) \]

\[ P(I = 1|F = \text{Good}) \]
4 Markov Decision Processes (MDPs) (25 points total).

Bob is doing a take-home exam. His only objective is to maximize the number of points he gets (without cheating, of course). He is in one of three states: Awake (A), Tired (T), or Sleeping (S). If he is Awake or Tired, he can work (w) or try to sleep (s). He gets 1 point if he works while Tired in a period, and 4 points if he works while Awake. If he is Sleeping or tries to sleep, he gets no points in that period.

If he is Sleeping, then with probability 1 he will be Awake the next period. If he is Tired and tries to sleep, he will be Sleeping with probability 1 the next period; if he is Tired and he works, he will be Tired with probability 1 the next period. If he is Awake and tries to sleep, he will be Sleeping with probability 1/2 and Awake with probability 1/2 the next period; if he is Awake and he works, he will be Tired with probability 1/2 and Awake with probability 1/2 the next period. The MDP is summarized in Figure 2.

a (10 points). Suppose there is a deadline for the take-home exam: Bob has $T = 3$ periods left before he must turn in the exam. There is no discounting. Use 3 iterations of value iteration (starting with values 0 for when there are no periods left) to compute both the optimal policy and the value of being in each state. Both of these should be a function of how many rounds $t$, with $1 \leq t \leq T$, are left. I.e., you should calculate $V_t(S)$ for every state $S$ and number of periods $t$ left, and state what the optimal action at that point is.
b (15 points). Now suppose that there is no deadline, but there is a discount factor $\gamma$. This can be interpreted as Bob being unable to keep track of time, and there being a $1 - \gamma$ chance he has to turn it in immediately at the end of each period. If $\gamma$ is close enough to 1 (i.e., the exam is not likely to end soon), what is the optimal policy (and why)? Give an exact formula for the value of being in each state (as a function of $\gamma$), assuming that $\gamma$ is close enough to 1.