Late-Term Exam Review

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Course Evaluations

- I want to stress that we take this seriously.

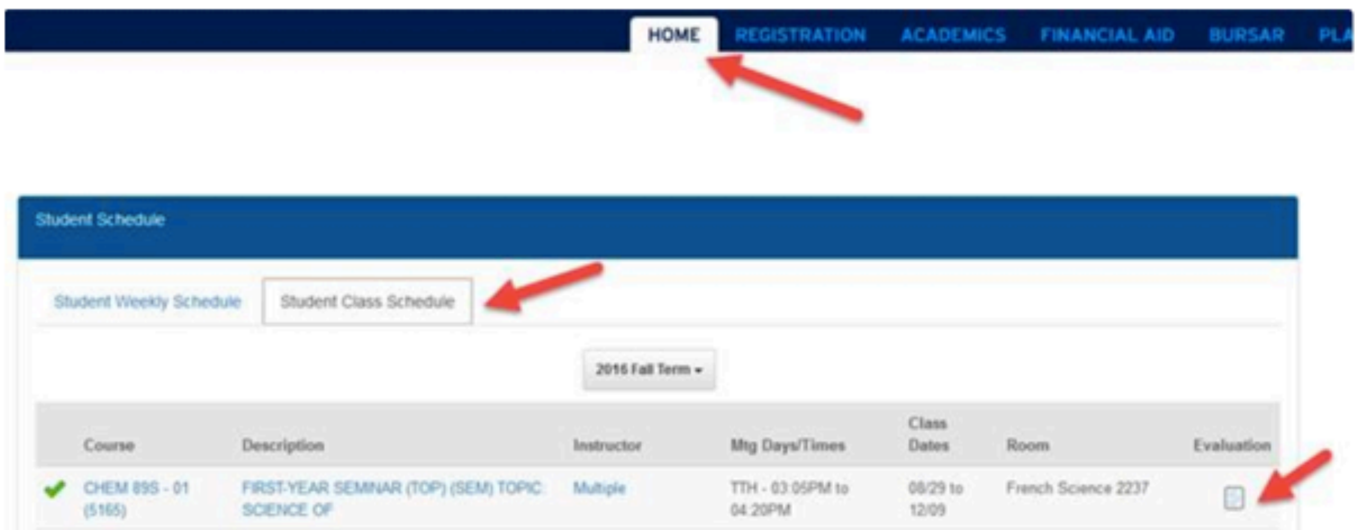
- Part of the reason this lab exists is in response to student feedback.

- Please let us know what you liked and did not like, what we should keep and what we can make better.

Course Evaluations

- Go to https://dukehub.duke.edu/

2. Click on the **Evaluation** icon (see image below) to begin the evaluation process. A course evaluation form will open up.

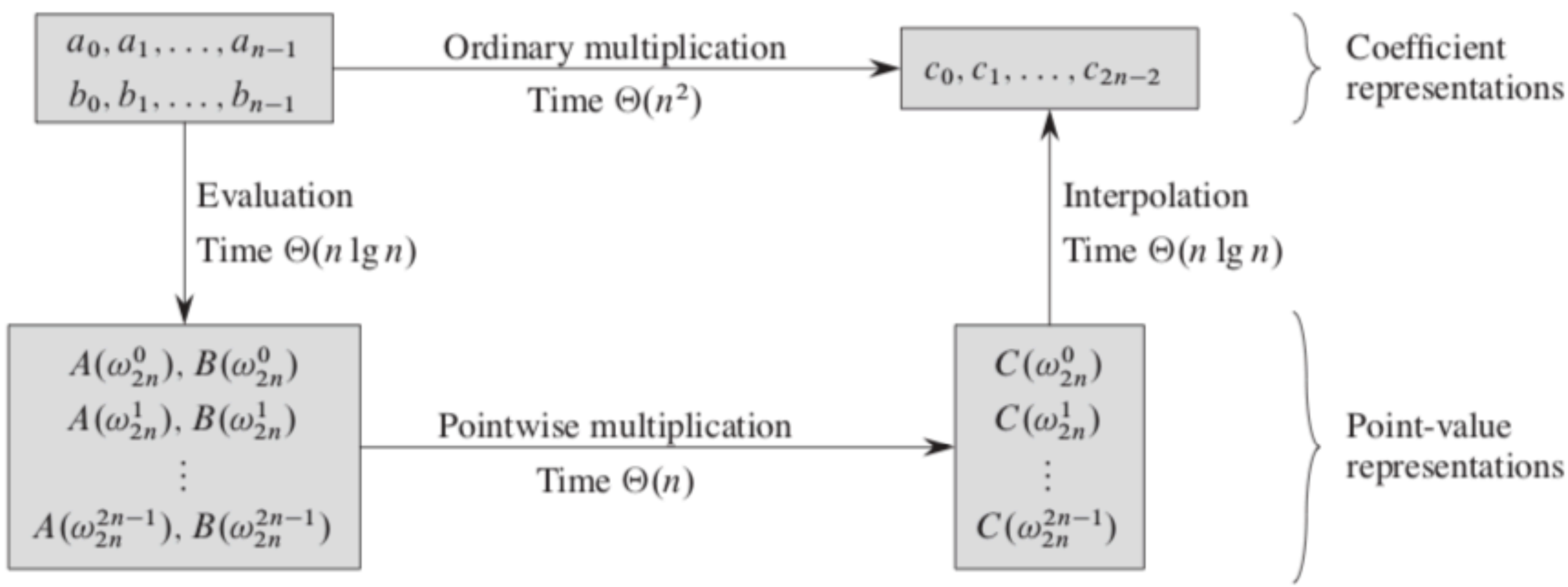


Late-Term Exam Format

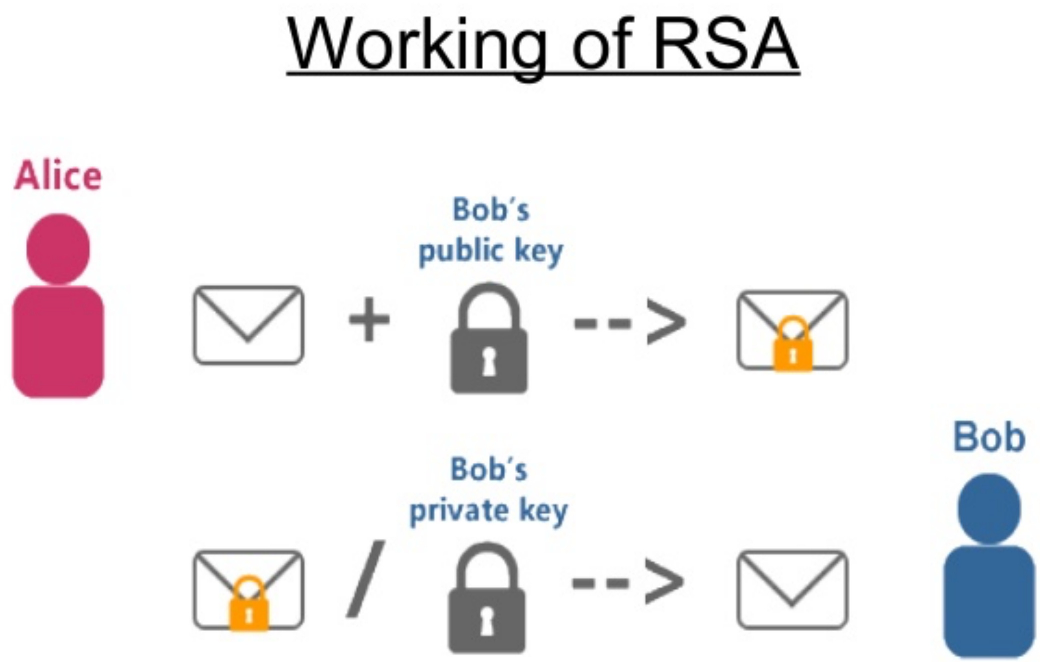
- 4 problems, each will ask you to write and analyze an algorithm
- 2 from graph theory
  - Connectivity: DFS, connected components, cycles, topological sort
  - Short paths: BFS, Dijkstra’s, Bellman-Ford
  - Spanning Trees: Greedy, Prim, Kruskal
- 2 from other topics from lecture
  - Polynomial multiplication and FFT
  - Number theory algorithms and RSA
  - Pattern matching
  - Computational Geometry
  - Dynamic Programming

Polynomial Multiplication and FFT

Chapter 30 Polynomials and the FFT



Public Key Cryptography



1. Select at random two LARGE prime numbers p and q (100-200 decimal digits).

2. Compute $n = pq$.

3. Select a small odd integer e relatively prime to $\phi(n) = (p-1)(q-1)$= number nontrivial factors of n

4. Compute d such that $ed = 1 \ mod \ \phi(n)$ (d exists and is unique!!!).

5. Publish the **public key** function $P_A(M) = M^e \ mod \ n$ (the pair (e, n)).

6. Keep secret the **secret key** function $S_A(C) = C^d \ mod \ n$.

Pattern Matching

- Input: Two strings $T[1\dots n]$ and $P[1\dots m]$, containing symbols from alphabet Σ.

  E.g. :

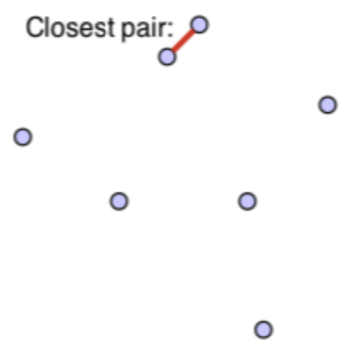
  - $\Sigma=\{a,b,\dots,z\}$
  - $T[1\dots 18]$="to be or not to be"
  - $P[1..2]$="be"

- Goal: find all "shifts" $0\leq s \leq n-m$ such that $T[s+1\dots s+m]=P$

  E.g. 3, 16

Computational Geometry

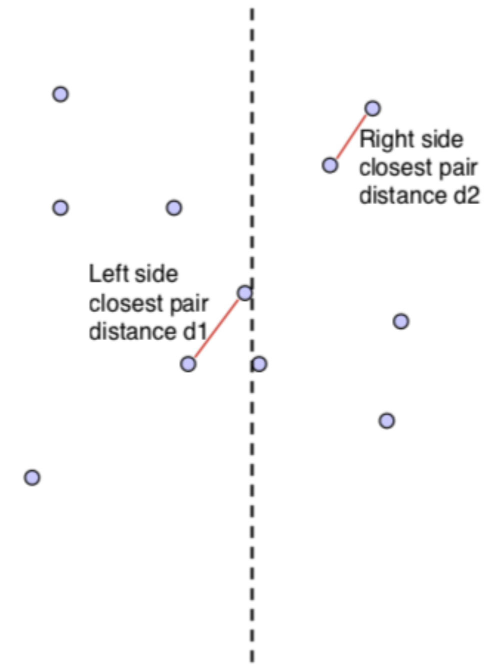
- Algorithms for geometric problems
- Applications: CAD, GIS, computer vision,.......
- E.g., the *closest pair* problem:
  - Given: a set of points $P=\{p_1 \ldots p_n\}$ in the plane, such that $p_i=(x_i,y_i)$
  - Goal: find a pair $p_i \neq p_j$ that minimizes $||p_i - p_j||$



$$||p-q|| = [(p_x-q_x)^2+(p_y-q_y)^2]^{1/2}$$

Computational Geometry

- Divide:
  - Compute the median of x-coordinates
  - Split the points into P_L and P_R, each of size $n/2$
- Conquer: compute the closest pairs for P_L and P_R
- Combine the results (the hard part)



Dynamic Programming

Optimal substructure

An optimal solution to a problem (instance) contains optimal solutions to subproblems.

Overlapping subproblems

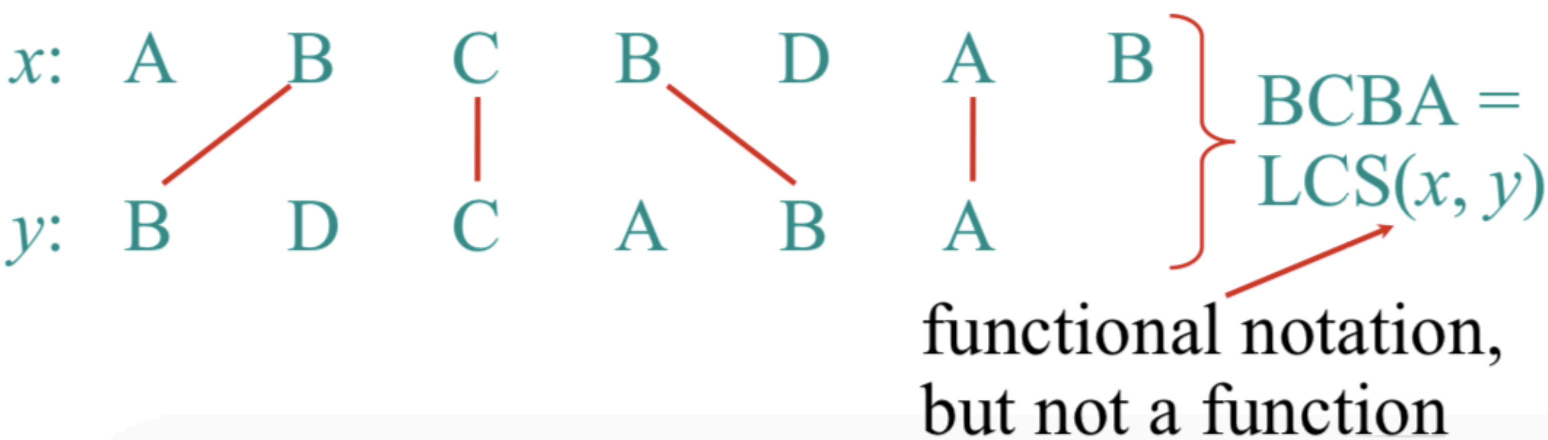
A recursive solution contains a "small" number of distinct subproblems repeated many times.

Dynamic Programming

Example: *Longest Common Subsequence (LCS)*

- Given two sequences $x[1\,..\,m]$ and $y[1\,..\,n]$, find a longest subsequence common to them both.

"a" *not* "the"



Dynamic Programming

Strategy: Consider ***prefixes*** of x and y.

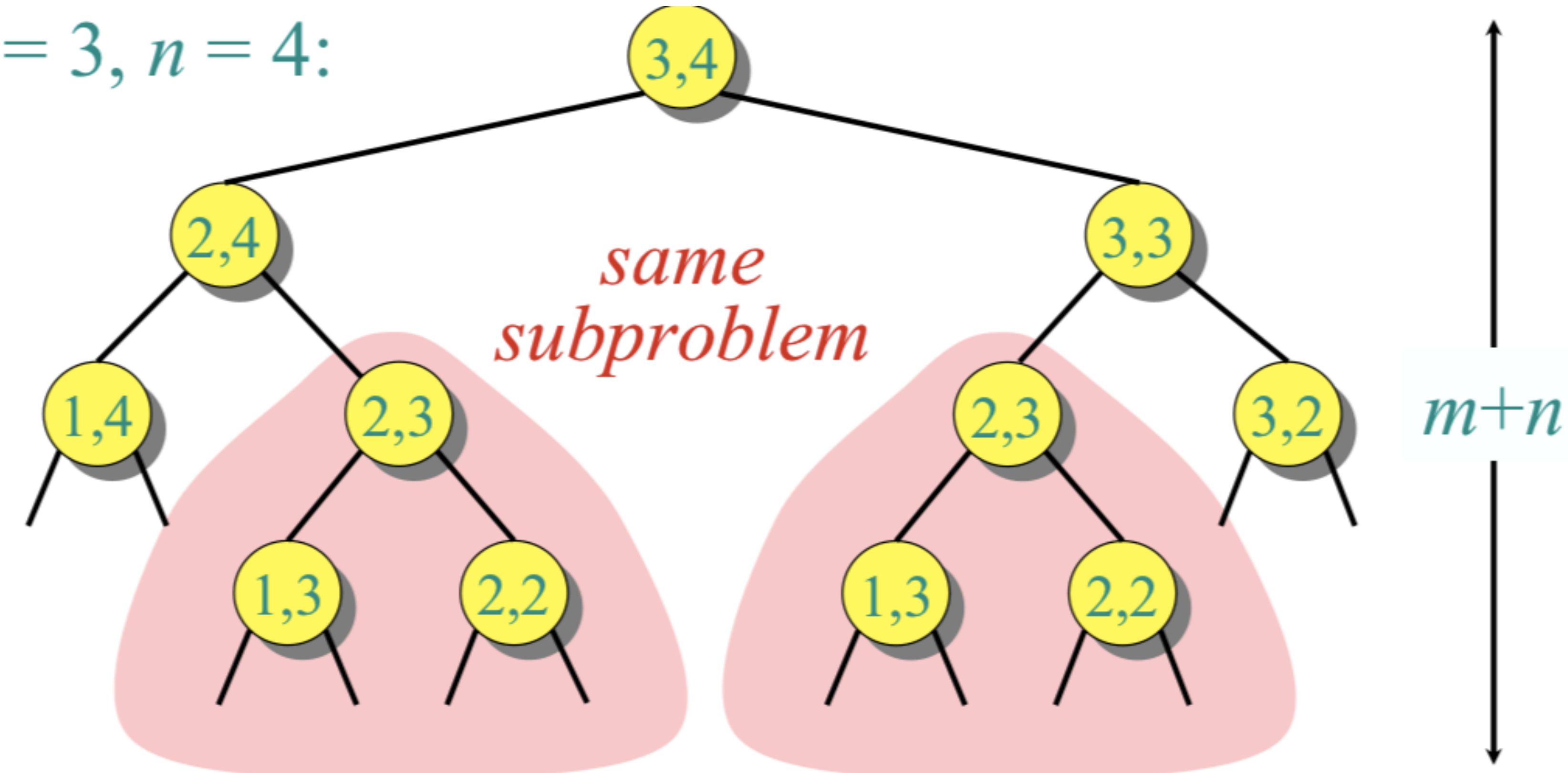
- Define $c[i, j] = |\mathrm{LCS}(x[1 \,..\, i], y[1 \,..\, j])|$.
- Then, $c[m, n] = |\mathrm{LCS}(x, y)|$.

Theorem.

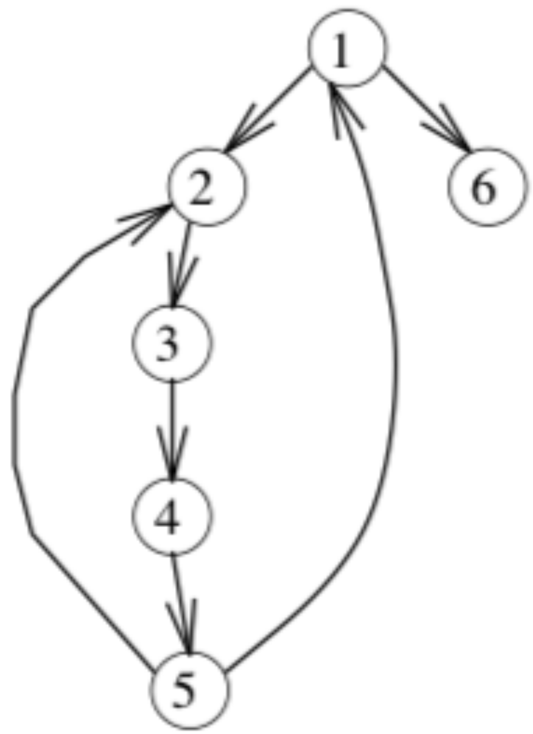
$$c[i, j] = \begin{cases} c[i-1, j-1] + 1 & \text{if } x[i] = y[j], \\ \max\{c[i-1, j], c[i, j-1]\} & \text{otherwise.} \end{cases}$$

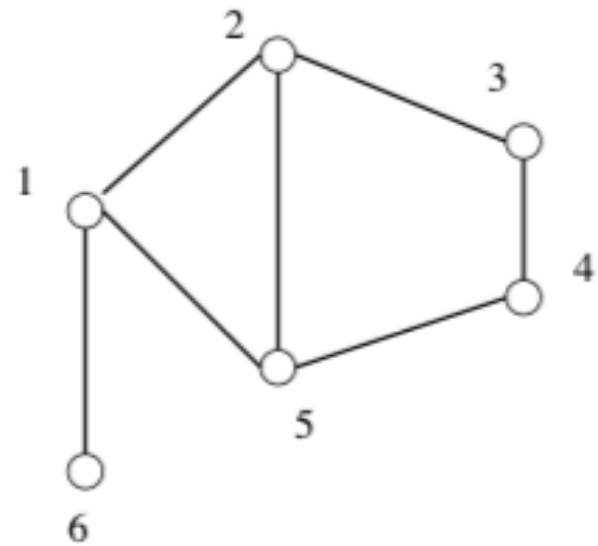
Dynamic Programming

$m = 3, n = 4$:



Graph Connectivity – Depth First Search

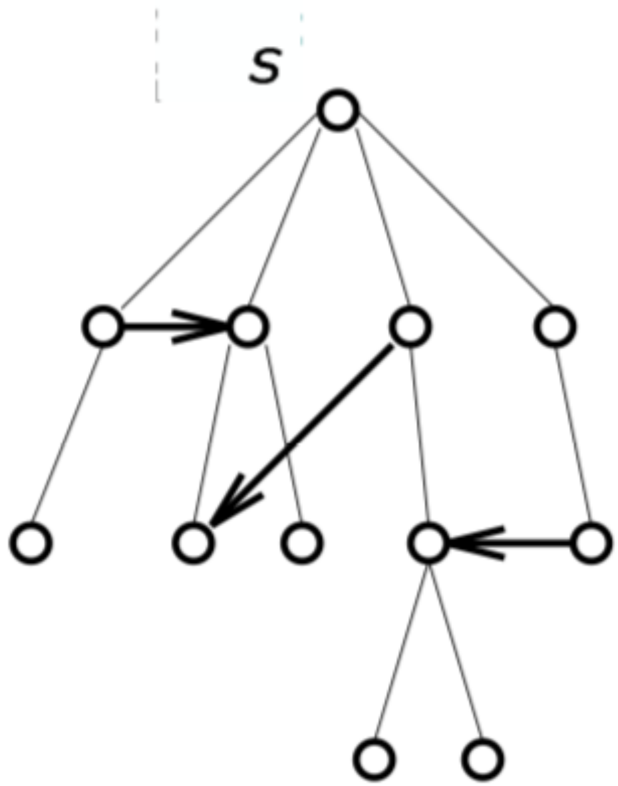




Graph Connectivity – Depth First Search

- **Runtime.**
  - $O(|V| + |E|)$ using adjacency lists.
  - $O(|V|^2)$ using adjacency matrix
  - In a dense graph, both are the same.
- **Applications**
  - Connectivity - "Does there exist a path from u to v?" Also, discovering connected components.
  - Cycle Detection – Just look for a "back" edge.
  - Topological Sort – Find a directed acyclic graph such that all edges are left to right (to do this, sort decreasing by finish time).

Short Paths – Breadth First Search



Runtime
- O(|V|+|E|)

Applications
- Shortest path in an unweighted graph
- Graph coloring / Testing for bipartite graph

Short Paths – Dijkstra's Algorithm

- Exchange a standard queue in breadth first search for a *priority queue* maintained on minimum distance so far.
- **Runtime**
  - O(|E|log(|V|)) with a binary heap
- **Application**
  - Shortest paths in *weighted* graphs with *no* negative edges

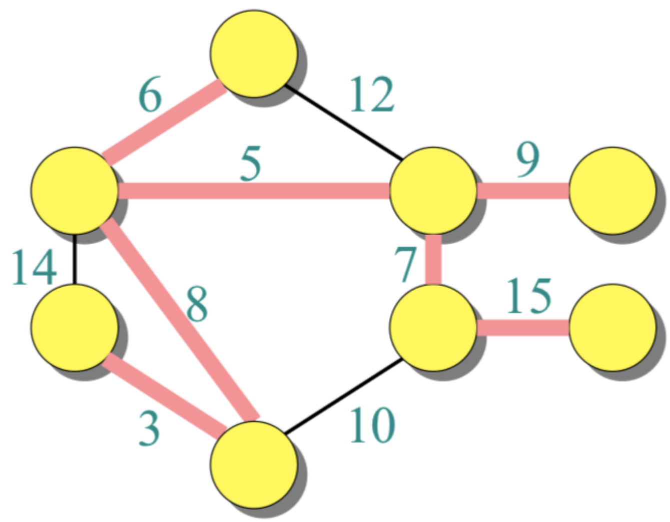
Short Paths – Bellman Ford

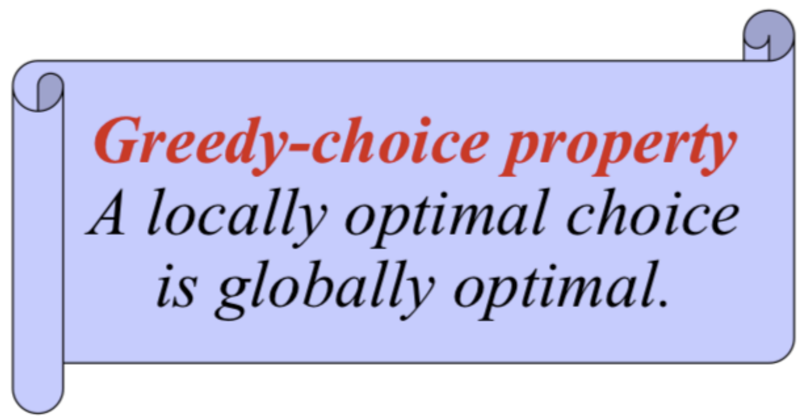
- Rather than making a clever exploration of the graph…
- Repeat |V|-1 times:
  - For every edge:
    - If that gives you a shorter path to some vertex, update.

- **Runtime**
  - O(|V||E|)
- **Application**
  - Shortest path in weighted graphs with negative edges but no negative cycles
  - Detecting negative cycles

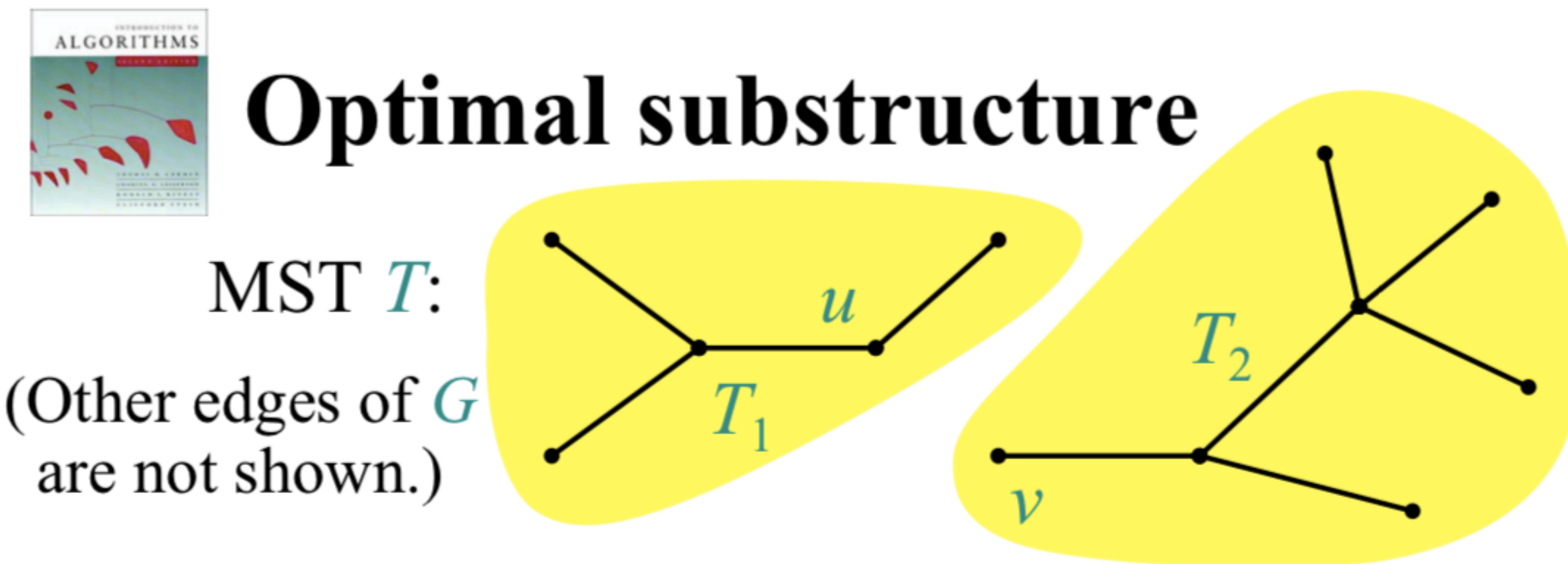
Greedy Algorithm – Spanning Trees

- A tree is a connected graph with no cycles.
- A spanning tree is a tree with every vertex in the graph.
- Minimum spanning trees have a greedy choice property.





Greedy Algorithm – Spanning Trees



- The MST of T_1 U T_2 just takes the "cheapest" edge between the two components.

Greedy Algorithm – Spanning Trees

- This intuition yields two algorithms for minimum spanning trees.
- **Prim's Algorithm** – Maintain a single tree / connected component. At each step include the vertex outside the current tree with the cheapest edge to the current tree.
- **Kruskal's Algorithm** – Maintain many different trees / connected components. At each step, merge any two components using the cheapest edge possible.
- **Runtime**
  - O(|E|log(|V|)) for both, but…
  - Prim's algorithm just needs a priority queue, Kruskal's algorithm needs a new disjoint set data structure for maintaining and merging components.

Questions?