Graph Definitions

Definition 1. *An* **undirected** *graph* G *is a pair* (V, E) *where*

- V *is the set of vertices,*
- $E \subseteq V^2$ *is the set of edges (unordered pairs)*

$$E \ = \ \{(u, v) \mid u, v \in V\}.$$

In a **directed** *graph the edges have directions (ordered pairs).*

A **weighted** *graph includes a weight function*

$$w : E \rightarrow R$$

attaching a value (weight) to each edge.

Definition 2. *A path in a graph* $G = (V, E)$ *is a sequence of vertices* $v_1, v_2, ..., v_k$ *such that for* $1 \leq i \leq k - 1$, $(v_i, v_{i+1}) \in E$.

Definition 3. *A cycle in a graph* $G = (V, E)$ *is a path* $v_1, v_2, ..., v_k$ *such that* $(v_k, v_1) \in E$.

Definition 4. *A tree is a graph with no cycles.*

Definition 5. *A graph* $H = (V', E')$ *is a* **subgraph** *of* $G = (V, E)$ *iff* $V' \subseteq V$ *and* $E' \subseteq E$. *A* **spanning** *subgraph if* $V' = V$.

Graph Representation

Adjacency List: A link list for each vertex. The list contains a pointer to each neighbor of the vertex. (and a weight for weighted graph.)

Total space $O(V + E)$.

Sequential (linear) access.

Adjacency Matrix: A $|V| \times |V|$ array, $A[i, j] = 1$ iff (i, j) is an edge in the graph, otherwise $A[i, j] = 0$. ($A[i, j] = w_{i,j}$ for weighted graph.)

Total space $O(V^2)$.

Random access.

Spanning Tree

Given a graph $G = (V, E)$ a **spanning tree** T in G is a subgraph of G that

- is connected;
- includes all the vertices of G;
- has no cycles;