Introduction to Cryptography

Li Yang, UTC

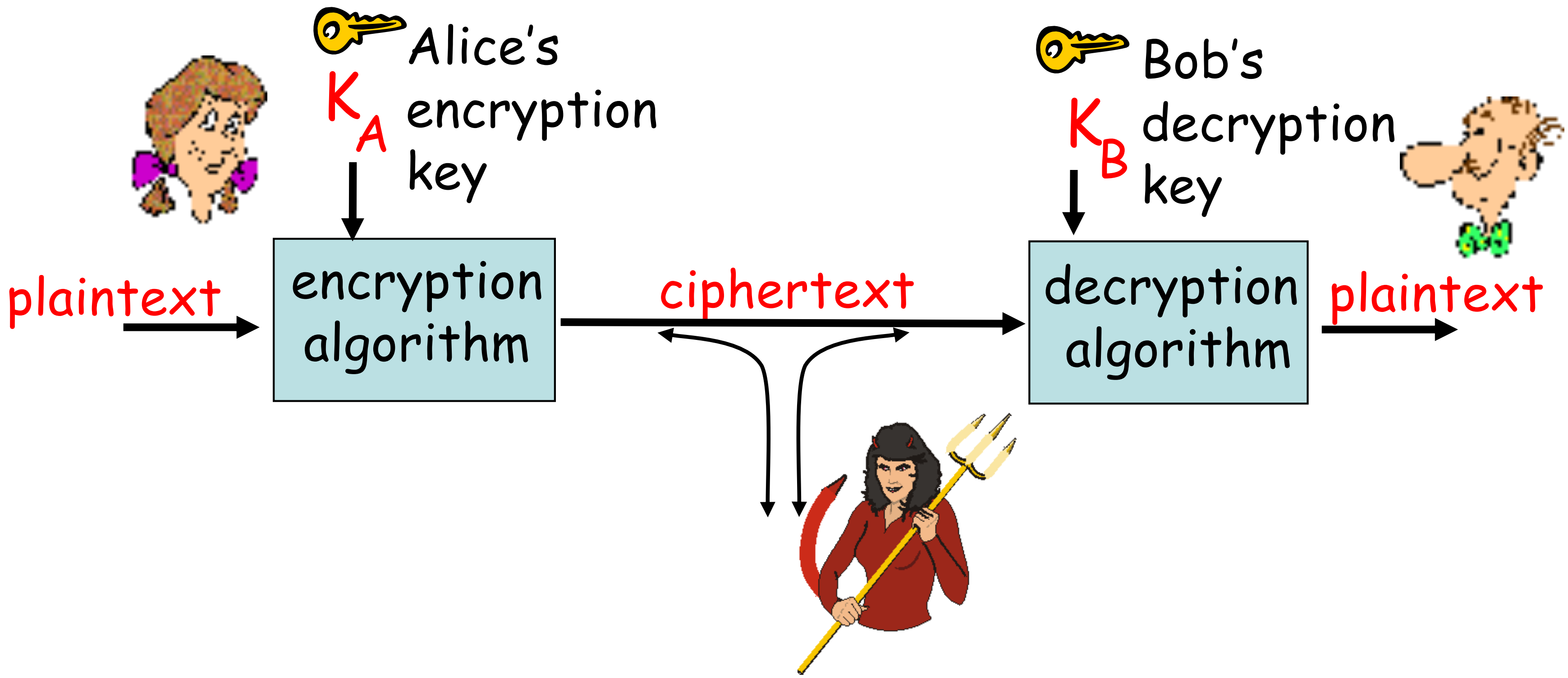
Summary

- Symmetric Encryption
- Public Encryption
- Digital Signature
- Key Distribution

Basic Terminology

- **plaintext** - the original message
- **ciphertext** - the coded message
- **cipher** - algorithm for transforming plaintext to ciphertext
- **key** - info used in cipher known only to sender/receiver
- **encipher (encrypt)** - converting plaintext to ciphertext
- **decipher (decrypt)** - recovering ciphertext from plaintext
- **cryptography** - study of encryption principles/methods
- **cryptanalysis (codebreaking)** - the study of principles/ methods of deciphering ciphertext *without* knowing key
- **cryptology** - the field of both cryptography and cryptanalysis

The language of cryptography



symmetric key crypto: sender, receiver keys *identical*

public-key crypto: encryption key *public*, decryption key *secret* (private)

Cryptography

- can characterize by:
  - type of encryption operations used
    - substitution / transposition / product
  - number of keys used
    - single-key or private / two-key or public
  - way in which plaintext is processed
    - block / stream

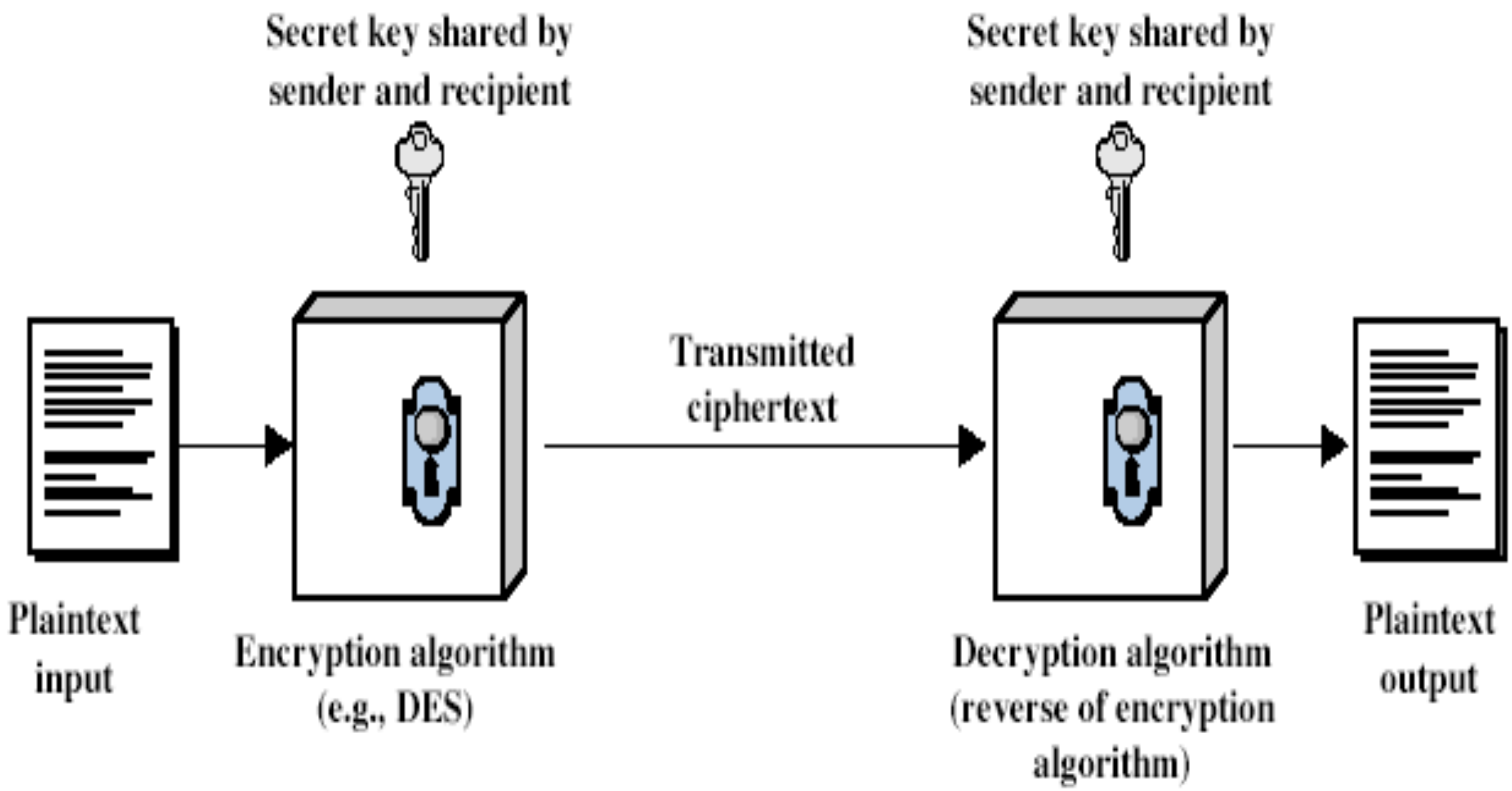
More Definitions

- **unconditional security**
  - no matter how much computer power is available, the cipher cannot be broken since the ciphertext provides insufficient information to uniquely determine the corresponding plaintext
- **computational security**
  - given limited computing resources (eg time needed for calculations is greater than age of universe), the cipher cannot be broken

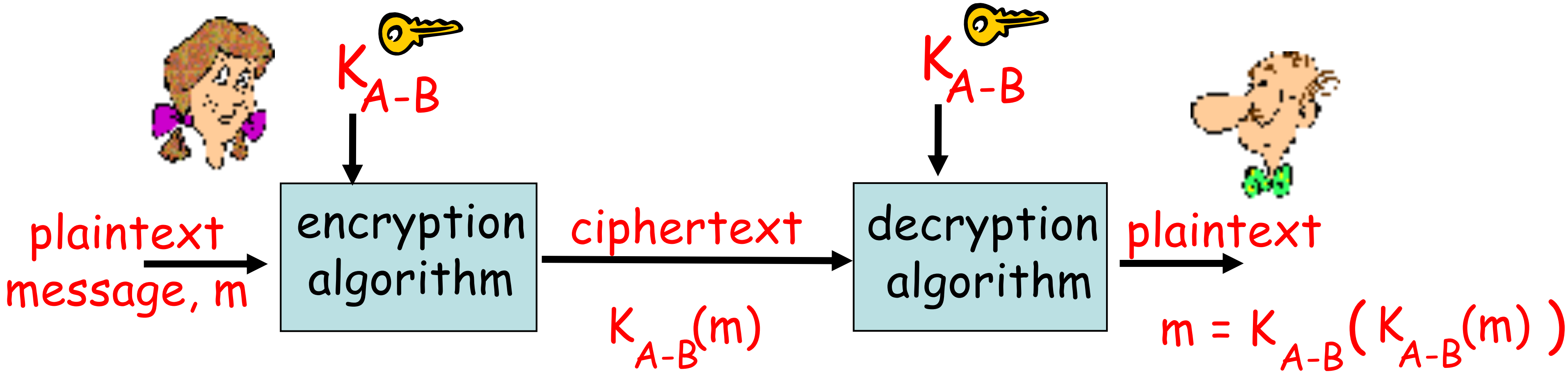
Symmetric Encryption

- or conventional / secret-key / single-key
- sender and recipient share a common key
- all classical encryption algorithms are private-key
- was only type prior to invention of public-key in 1970’s

Symmetric Cipher Model



Symmetric Key Cryptography



symmetric key crypto: Bob and Alice share know same (symmetric) key: K_{A-B}

- e.g., key is knowing substitution pattern in mono alphabetic substitution cipher

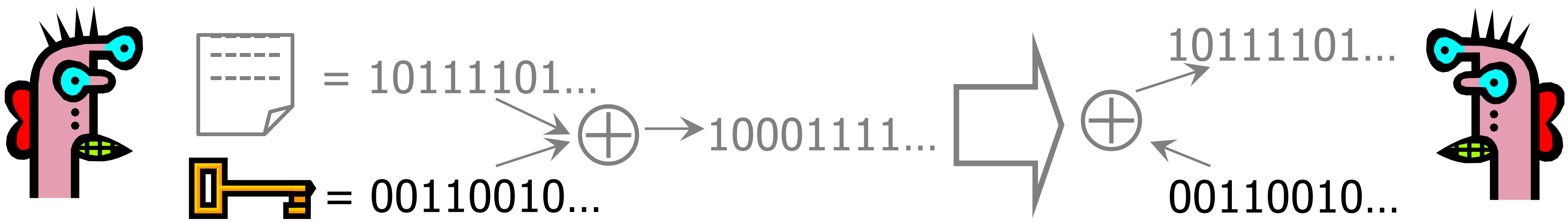
Requirements

- two requirements for secure use of symmetric encryption:
  - a strong encryption algorithm
  - a secret key known only to sender / receiver

    $Y = E_K(X)$

    $X = D_K(Y)$
- assume encryption algorithm is known
- implies a secure channel to distribute key

Simple Idea: One-Time Pad



Key is a never-repeating bit sequence as long as plaintext

Encrypt by bitwise XOR of plaintext and key:
ciphertext = plaintext ⊕ key

Decrypt by bitwise XOR of ciphertext and key:
ciphertext ⊕ key =
(plaintext ⊕ key) ⊕ key =
plaintext ⊕ (key ⊕ key) =
plaintext

Cipher achieves perfect secrecy if and only if there are as many possible keys as possible plaintexts, and every key is equally likely (Claude Shannon's result)

Advantages of One-Time Pad

- Easy to compute
  - Encryption and decryption are the same operation
  - Bitwise XOR is very cheap to compute
- As secure as possible
  - Given a ciphertext, all plaintexts are equally likely, regardless of attacker's computational resources
  - …as long as the key sequence is truly random
    - True randomness is expensive to obtain in large quantities
  - …as long as each key is same length as plaintext
    - But how does the sender communicate the key to receiver?

Problems with One-Time Pad

- Key must be as long as plaintext
  - Impractical in most realistic scenarios
  - Still used for diplomatic and intelligence traffic
- Does not guarantee integrity
  - One-time pad only guarantees confidentiality
  - Attacker cannot recover plaintext, but can easily change it to something else
- Insecure if keys are reused
  - Attacker can obtain XOR of plaintexts

Summary

- Symmetric encryption
- Public encryption
- Digital Signature
- Key distribution

Private-Key Cryptography

- traditional **private/secret/single key** cryptography uses **one** key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is **symmetric**, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

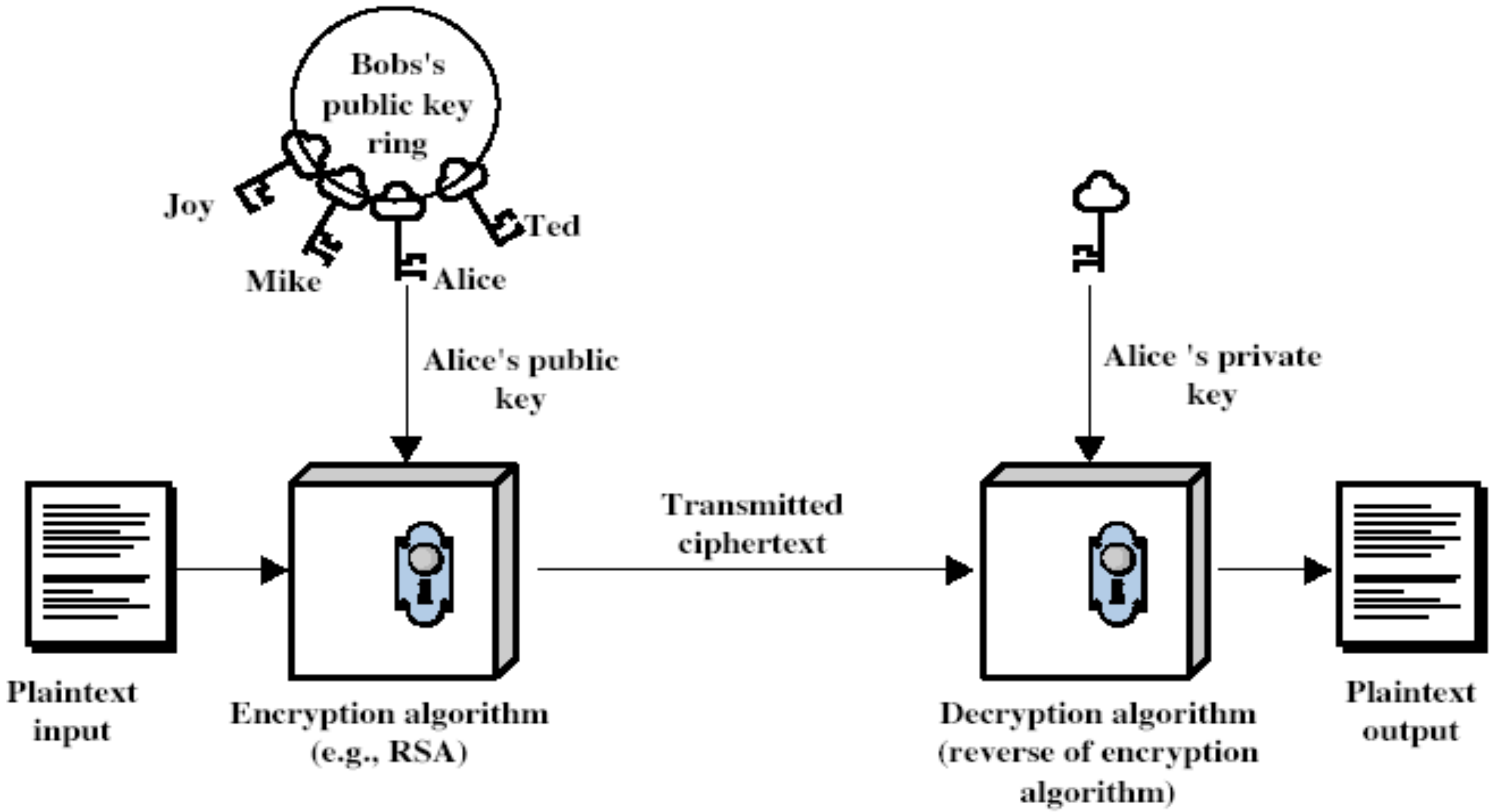
Public-Key Cryptography

- probably most significant advance in the 3000 year history of cryptography
- uses **two** keys – a public & a private key
- **asymmetric** since parties are **not** equal
- uses clever application of number theoretic concepts to function
- complements **rather than** replaces private key crypto

Public-Key Cryptography

- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - a **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures**
  - a **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures**
- is **asymmetric** because
  - those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures

Public-Key Cryptography



Public-Key Characteristics

- Public-Key algorithms rely on two keys with the characteristics that it is:
  - computationally infeasible to find decryption key knowing only algorithm & encryption key
  - computationally easy to en/decrypt messages when the relevant (en/decrypt) key is known
  - either of the two related keys can be used for encryption, with the other used for decryption (in some schemes)

Public-Key Cryptosystems

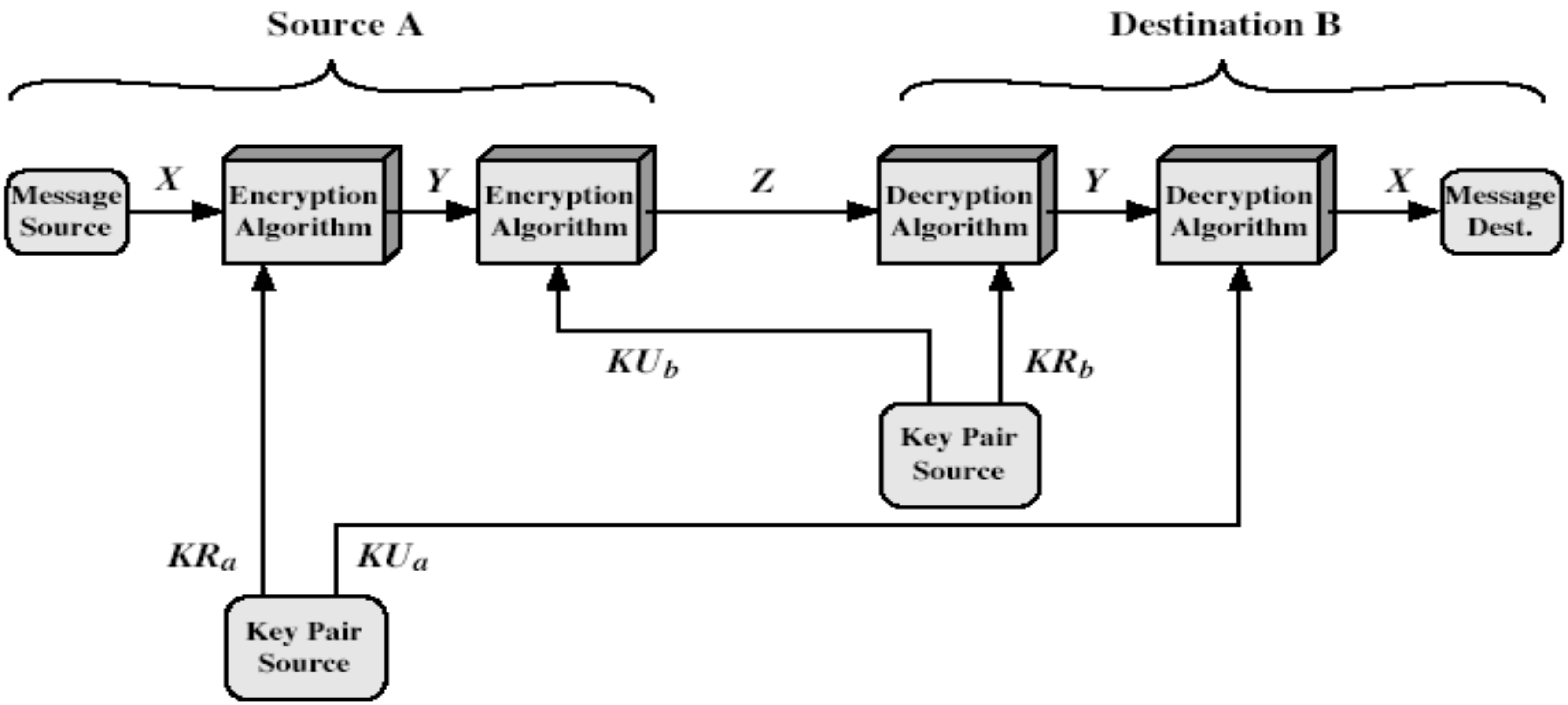


Figure 9.4 Public-Key Cryptosystem: Secrecy and Authentication

Public-Key Applications

- can classify uses into 3 categories:
  - **encryption/decryption** (provide secrecy)
  - **digital signatures** (provide authentication)
  - **key exchange** (of session keys)
- some algorithms are suitable for all uses, others are specific to one

Security of Public Key Schemes

- like private key schemes brute force **exhaustive search** attack is always theoretically possible
- but keys used are too large (>512bits)
- security relies on a **large enough** difference in difficulty between **easy** (en/decrypt) and **hard** (cryptanalysis) problems
- more generally the **hard** problem is known, its just made too hard to do in practise
- requires the use of **very large numbers**
- hence is **slow** compared to secret key schemes

Public key encryption algorithms

Requirements:

1. need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$

2. given public key K_B^+, it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

RSA: Choosing keys

1. Choose two large prime numbers *p, q.* (e.g., 1024 bits each)

2. Compute $n = pq$, $z = (p-1)(q-1)$

3. Choose e (with $e<n$) that has no common factors with z. (e, z are "relatively prime").

4. Choose d such that $ed-1$ is exactly divisible by z. (in other words: $ed \bmod z = 1$).

5. *Public* key is $\underbrace{(n,e)}_{K_B^+}$. *Private* key is $\underbrace{(n,d)}_{K_B^-}$.

RSA: Encryption, decryption

0. Given (n,e) and (n,d) as computed above

1. To encrypt bit pattern, m, compute
   $c = m^e \bmod n$ (i.e., remainder when m^e is divided by n)

2. To decrypt received bit pattern, c, compute
   $m = c^d \bmod n$ (i.e., remainder when c^d is divided by n)

Magic happens!

$$m = (\underbrace{m^e \bmod n}_{c})^d \bmod n$$

RSA example:

Bob chooses $p=5$, $q=7$. Then $n=35$, $z=24$.
$e=5$ (so e, z relatively prime).
$d=29$ (so $ed-1$ exactly divisible by z.

encrypt:

letter	m	m^e	$c = m^e \bmod n$
l	12	1524832	17

decrypt:

c	c^d	$m = c^d \bmod n$	letter
17	481968572106750915091411825223071697	12	l

RSA: Why is that $m = (m^e \bmod n)^d \bmod n$

Useful number theory result: If p,q prime and $n = pq$, then:

$$x^y \bmod n = x^{y \bmod (p-1)(q-1)} \bmod n$$

$$(m^e \bmod n)^d \bmod n = m^{ed} \bmod n$$

$$= m^{ed \bmod (p-1)(q-1)} \bmod n$$

(using number theory result above)

$$= m^1 \bmod n$$

(since we chose ed to be divisible by $(p-1)(q-1)$ with remainder 1)

$$= m$$

RSA: another important property

The following property will be *very* useful later:

$$\underbrace{K_B^-(K_B^+(m))}_{\substack{\text{use public key} \\ \text{first, followed} \\ \text{by private key}}} = m = \underbrace{K_B^+(K_B^-(m))}_{\substack{\text{use private key} \\ \text{first, followed} \\ \text{by public key}}}$$

Result is the same!

Summary

- Symmetric encryption
- Public encryption
- Digital Signature
- Key distribution

Digital Signatures

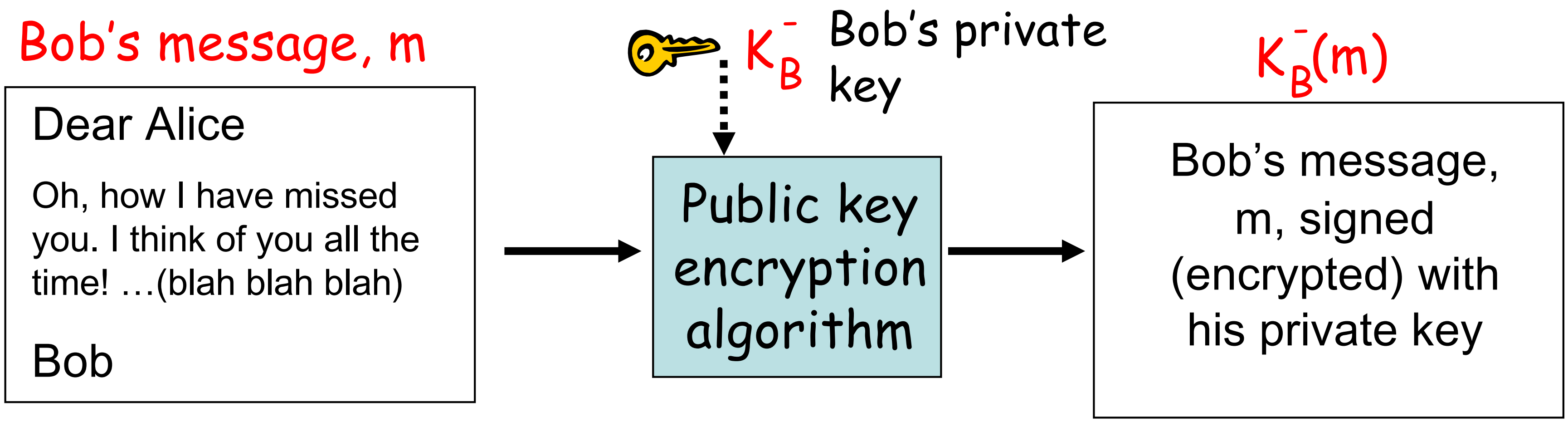
Cryptographic technique analogous to hand-written signatures.

- sender (Bob) digitally signs document, establishing he is document owner/creator.
- verifiable, nonforgeable: recipient (Alice) can prove to someone that Bob, and no one else (including Alice), must have signed document

Digital Signatures

Simple digital signature for message m:

- Bob signs m by encrypting with his private key K_B, creating "signed" message, $K_B(m)$



Digital Signatures (more)

- Suppose Alice receives msg m, digital signature $K_B^-(m)$
- Alice verifies m signed by Bob by applying Bob's public key K_B^+ to $K_B^-(m)$ then checks $K_B^+(K_B^-(m)) = m$.
- If $K_B^+(K_B^-(m)) = m$, whoever signed m must have used Bob's private key.

Alice thus verifies that:

✓ Bob signed m.

✓ No one else signed m.

✓ Bob signed m and not m'.

Non-repudiation:

✓ Alice can take m, and signature $K_B^-(m)$ to court and prove that Bob signed m.

Internet checksum: poor crypto hash function

Internet checksum has some properties of hash function:

- ✓ produces fixed length digest (16-bit sum) of message
- ✓ is many-to-one

But given message with given hash value, it is easy to find another message with same hash value:

message	ASCII format
I O U 1	**49 4F 55 31**
0 0 . 9	**30 30 2E 39**
9 B O B	**39 42 D2 42**
	B2 C1 D2 AC

message	ASCII format
I O U 9	**49 4F 55 39**
0 0 . 1	**30 30 2E 31**
9 B O B	**39 42 D2 42**
	B2 C1 D2 AC

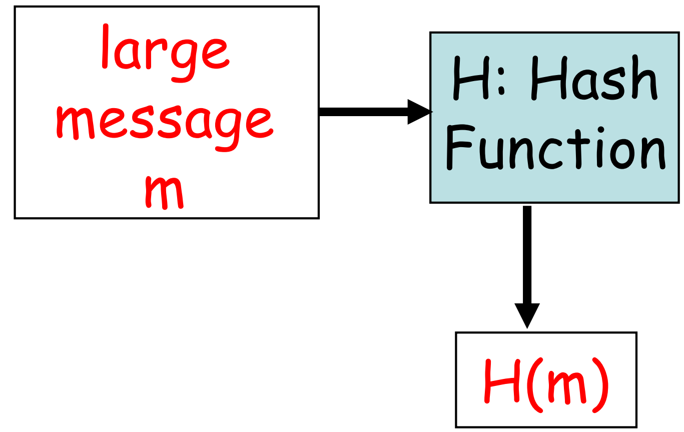
different messages but identical checksums!

Message Digests

Computationally expensive to public-key-encrypt long messages

Goal: fixed-length, easy- to-compute digital “fingerprint”

- apply hash function H to *m*, get fixed size message digest, *H(m).*



Hash function properties:

- many-to-1
- produces fixed-size msg digest (fingerprint)
- given message digest x, computationally infeasible to find m such that x = H(m)

Digital signature = signed message digest

Bob sends digitally signed message:

Alice verifies signature and integrity of digitally signed message: