Introduction to Tiling Assembly

Ho-Lin Chen

Nov 8 2005

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Self-Assembly

- Self-Assembly is the process by which *simple* objects *autonomously* assemble into complexes.
  - Geometry, dynamics, combinatorics are all important
  - Inorganic: Crystals, supramolecules
  - Organic: Proteins, DNA, cells, organisms
- Goals: Understand self-assembly, design self-assembling systems
  - A key problem in nano-technology, molecular robotics, molecular computation

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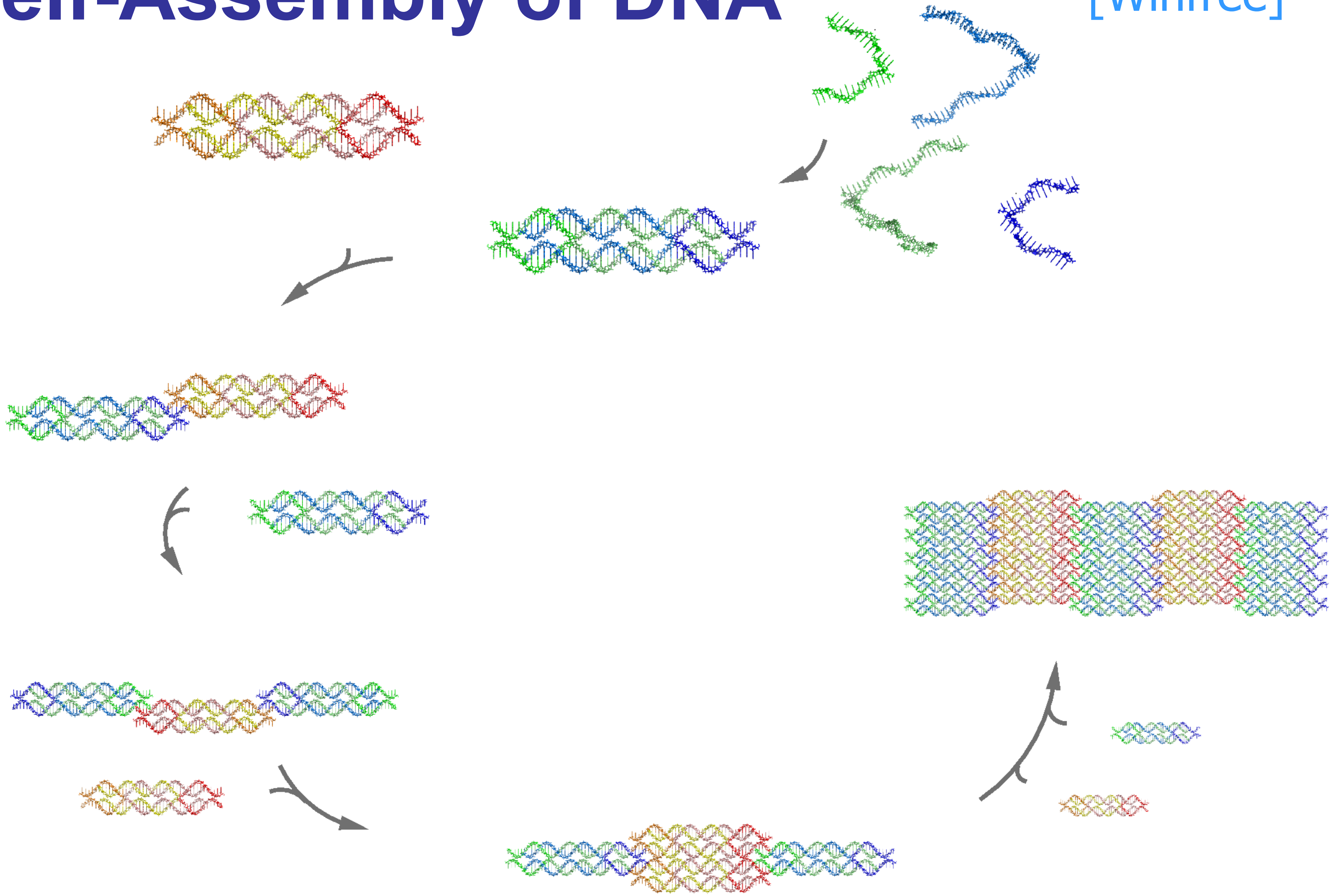
Applications of Self-Assembly

- Building blocks of nano-machines.
- DNA computing.
- Small electrical devices such as FLASH memory. [Black et. Al.’ 03]
- Nanostructures which “steer” light in the same way computer chips steer electrons.
  [Percec et. Al.’ 03]

Self-Assembly of DNA

[Winfree]



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Abstract System Model

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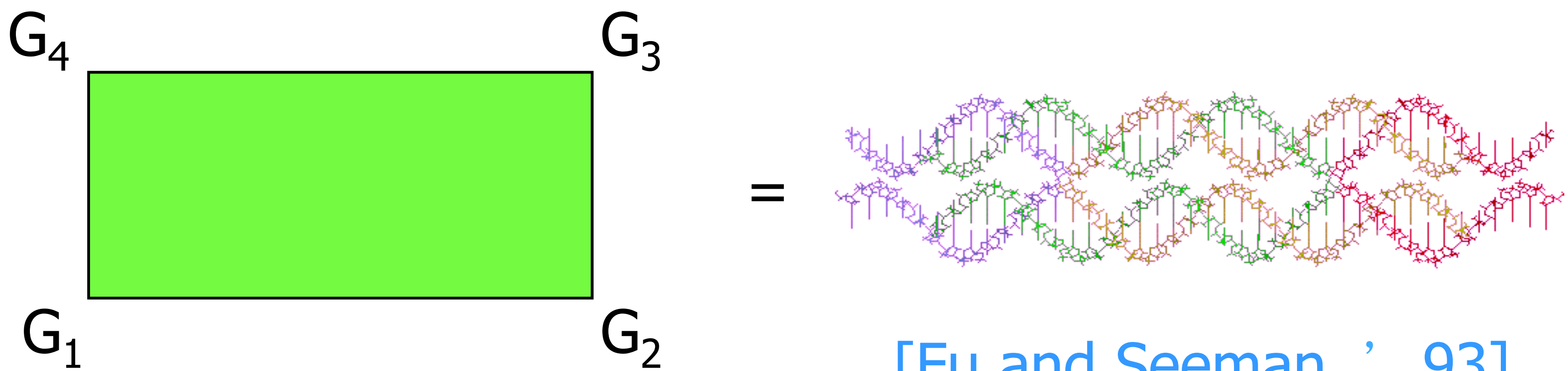
DNA Tiles



[Fu and Seeman, ' 93]

Glues = sticky ends
Tiles = molecules

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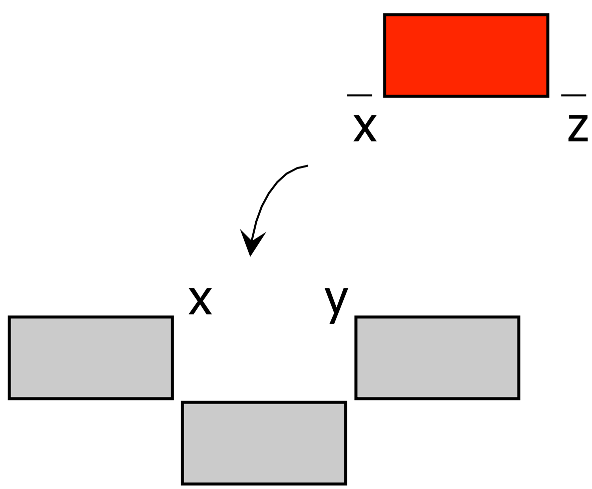
abstract Tile Assembly Model:

[Rothemund, Winfree, ’ 2000]

Temperature: A positive integer. (Usually 1 or 2)

A set of tile types: Each tile is an oriented rectangle with glues on its corners. Each glue has a non-negative strength (0, 1 or 2).

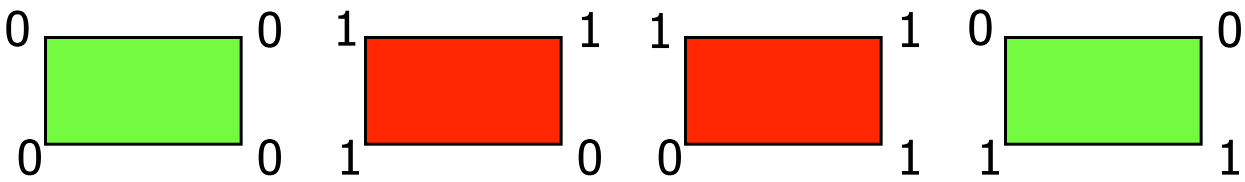
An initial assembly (seed).



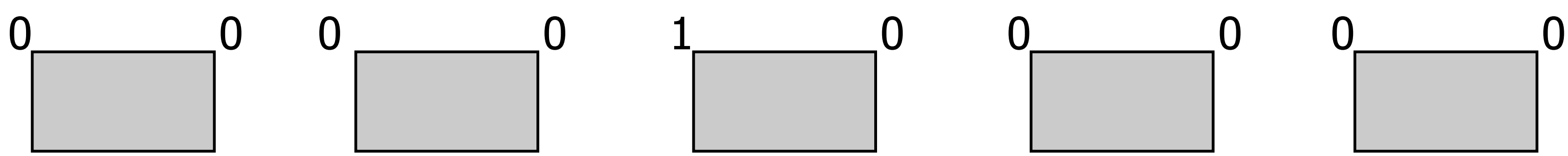
A tile can attach to an assembly iff the combined strength of the “matched glues” is greater or equal than the temperature.

Example: Sierpinski System

[Winfree, ’96]



T=2



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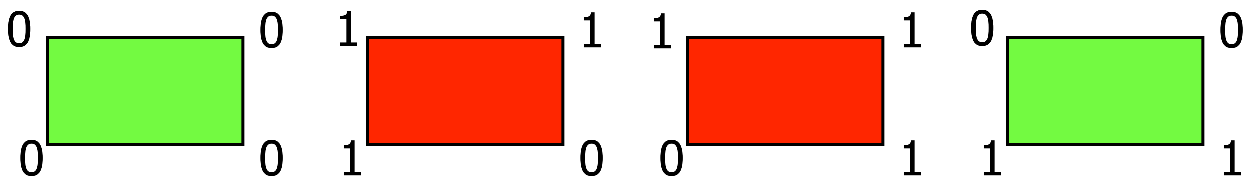

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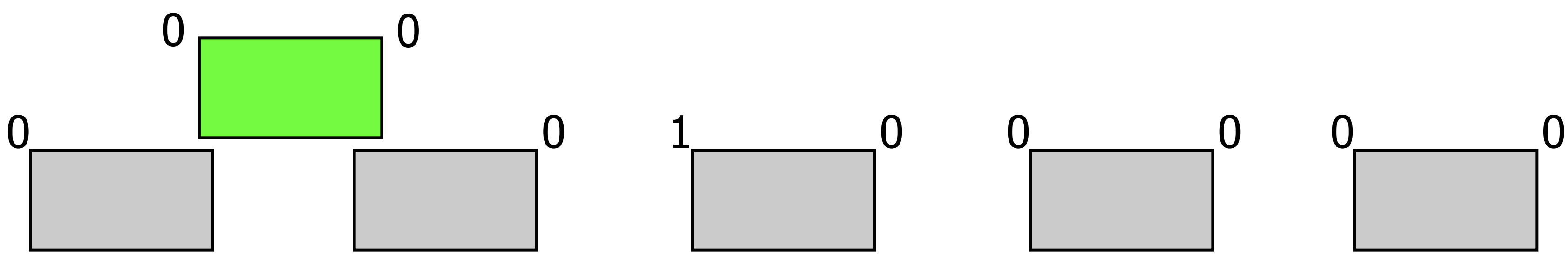

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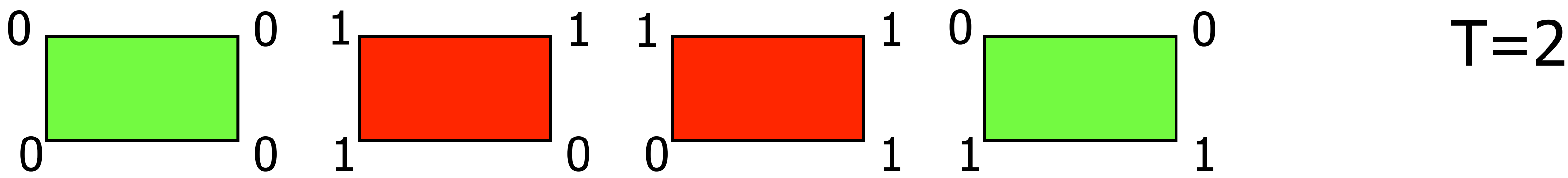

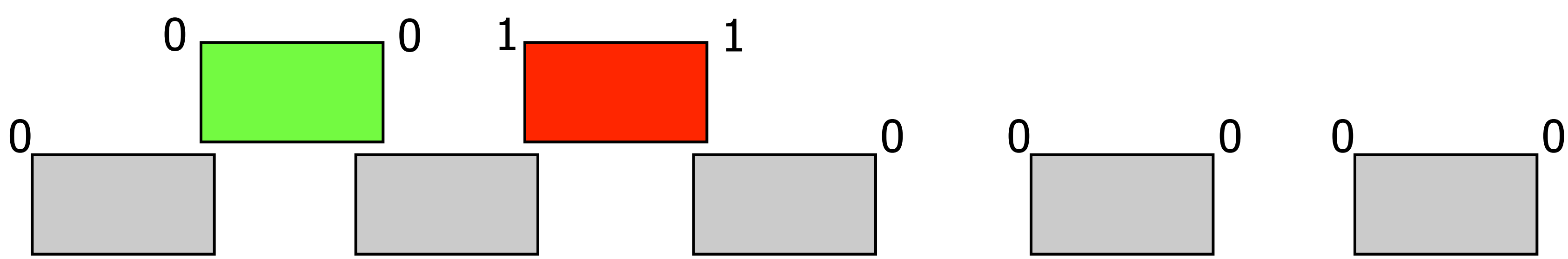

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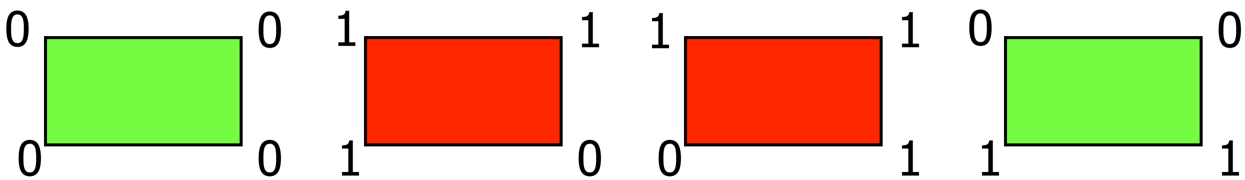

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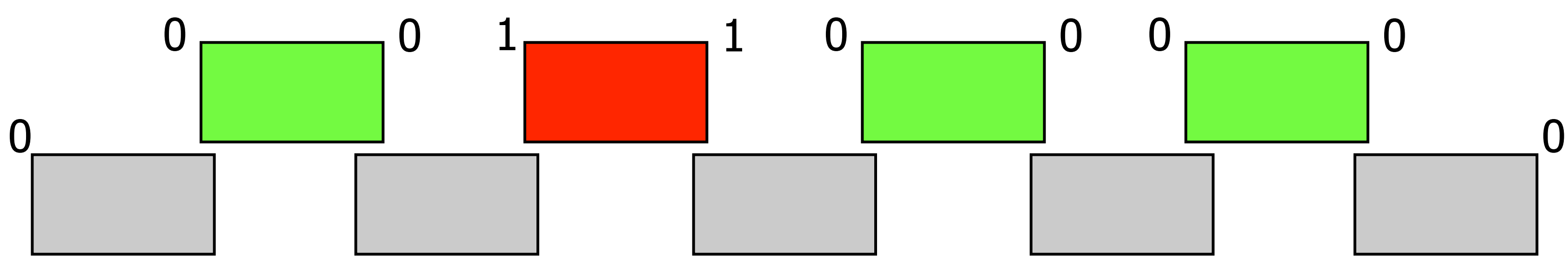

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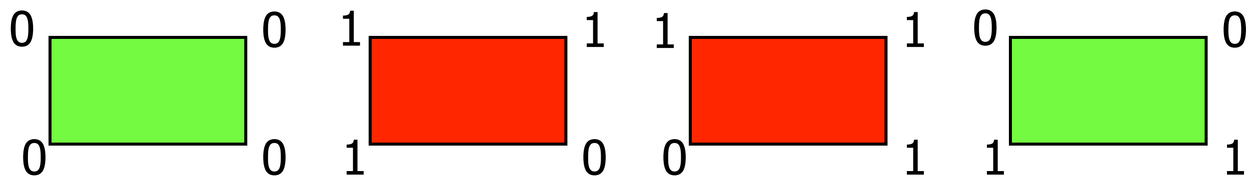

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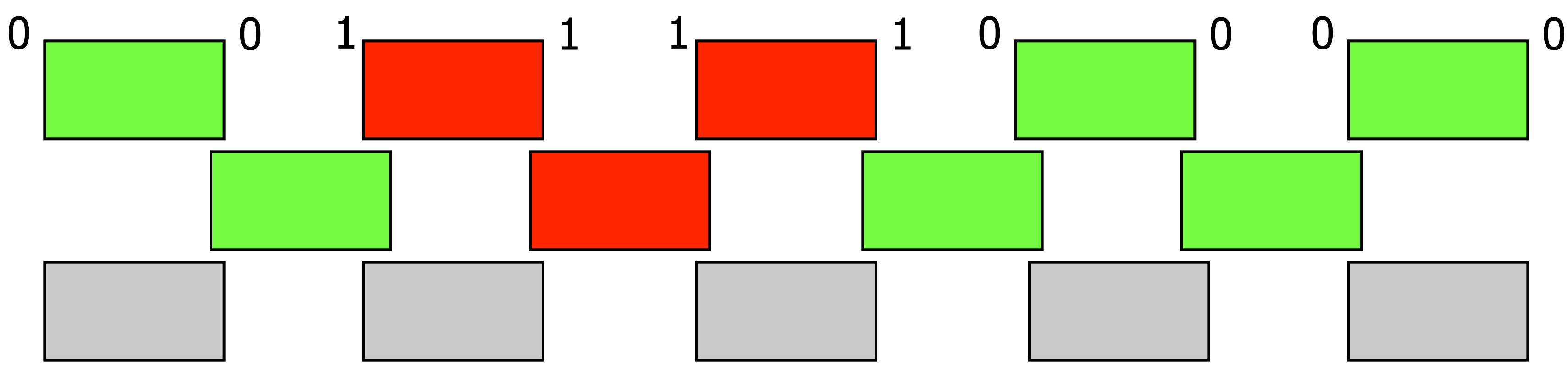

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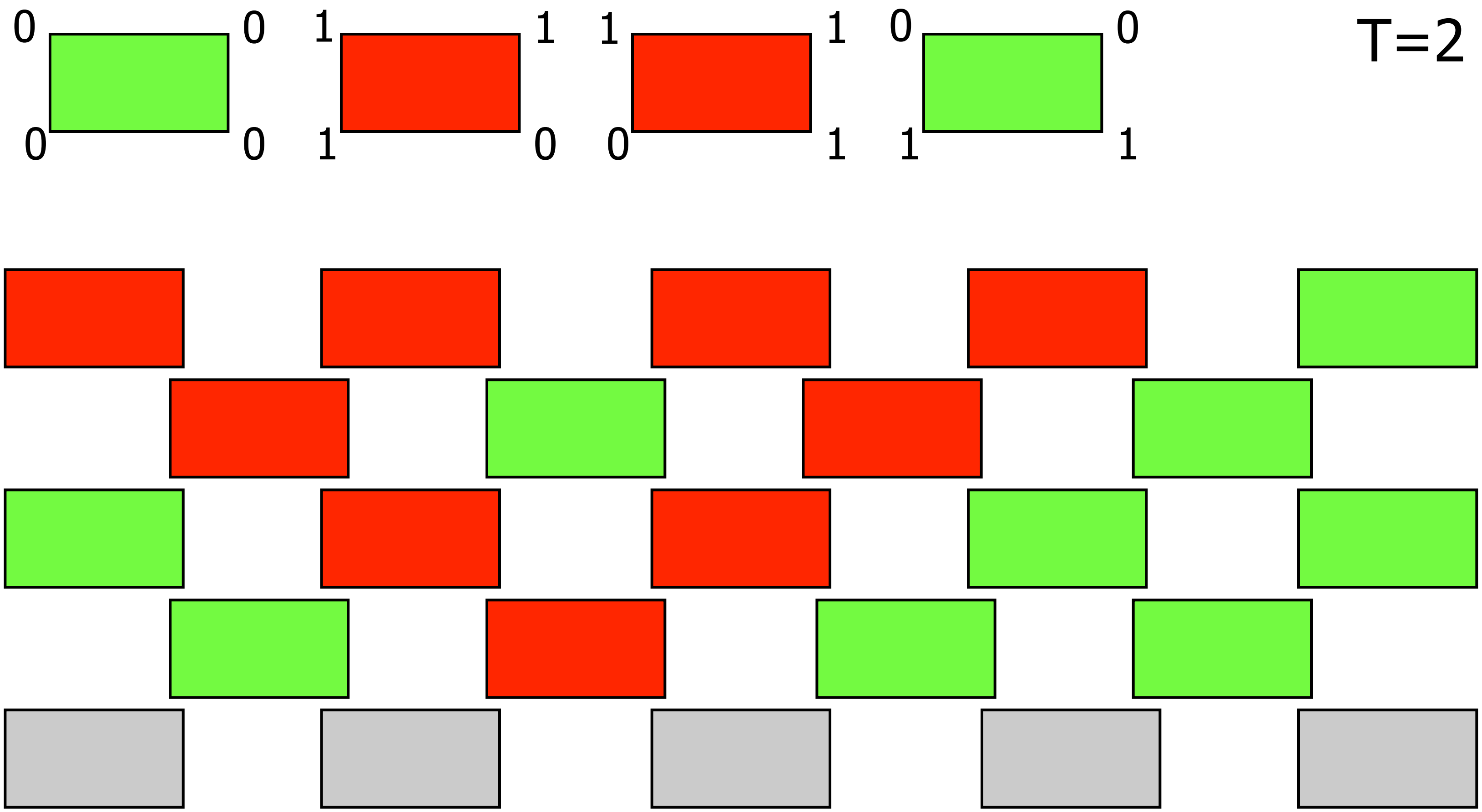
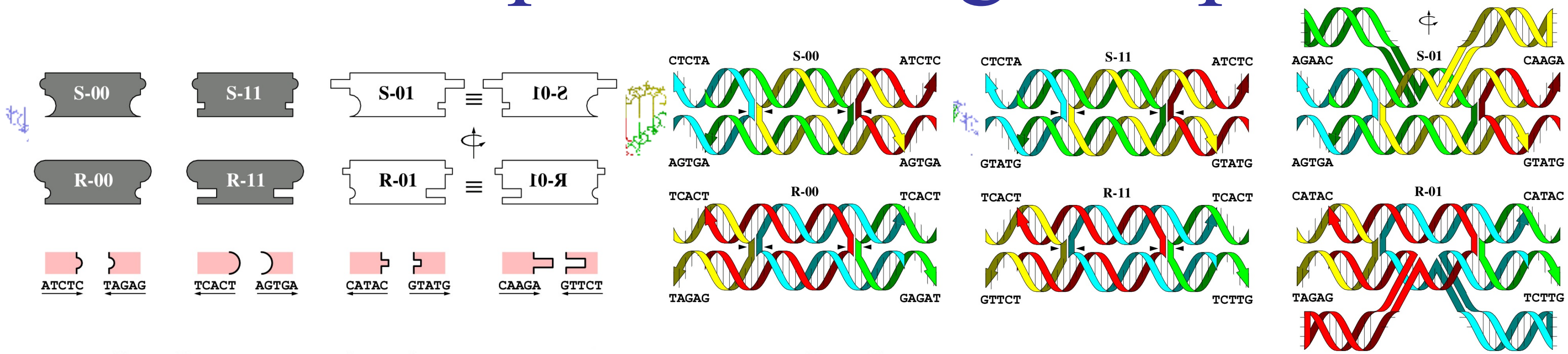
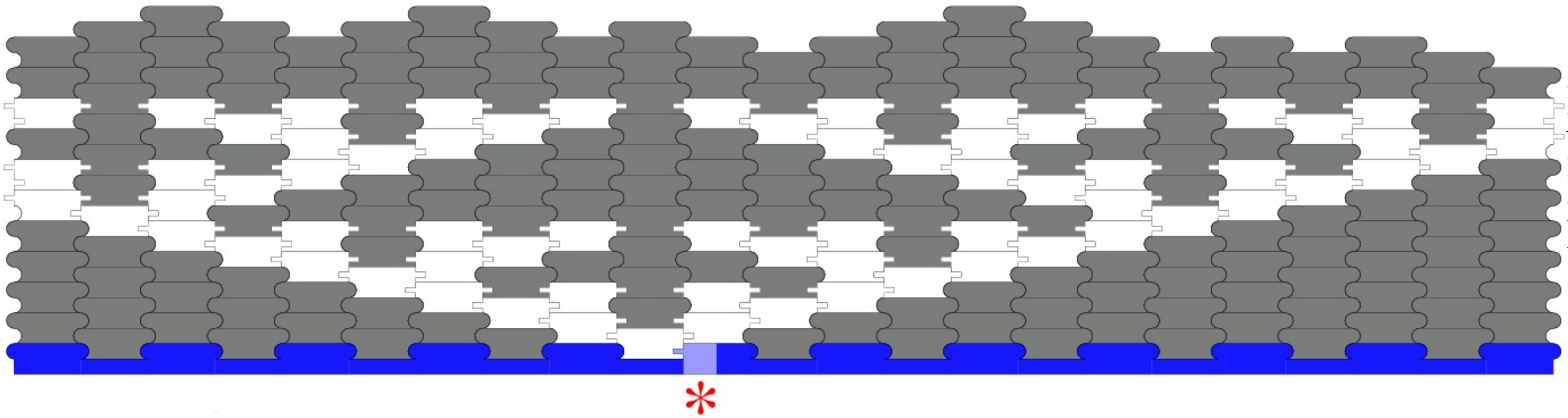
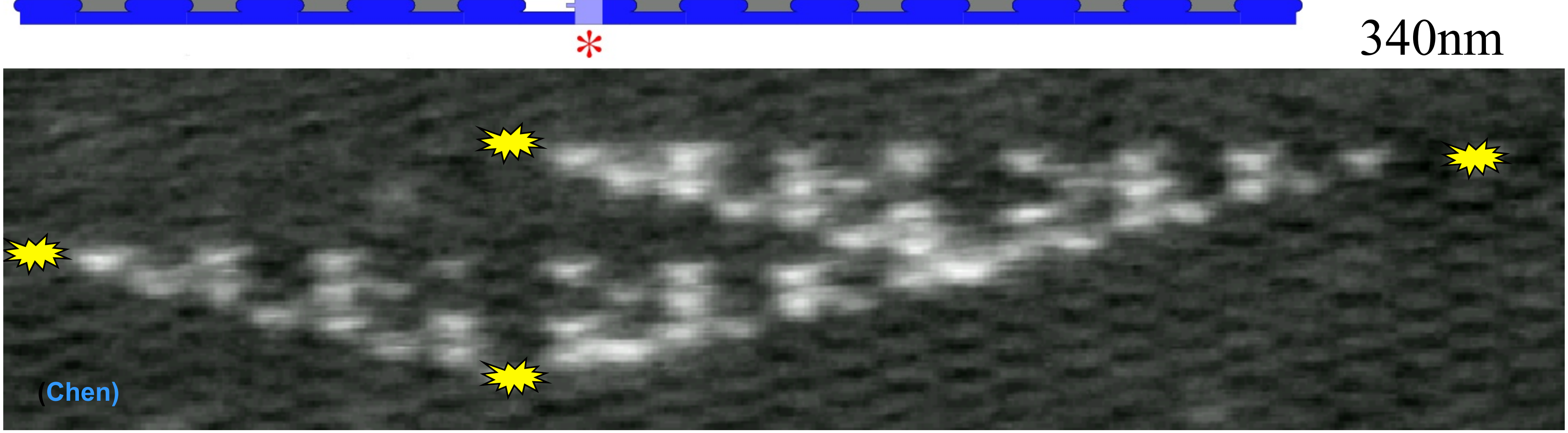


DAO-E Sierpinski triangle experiments





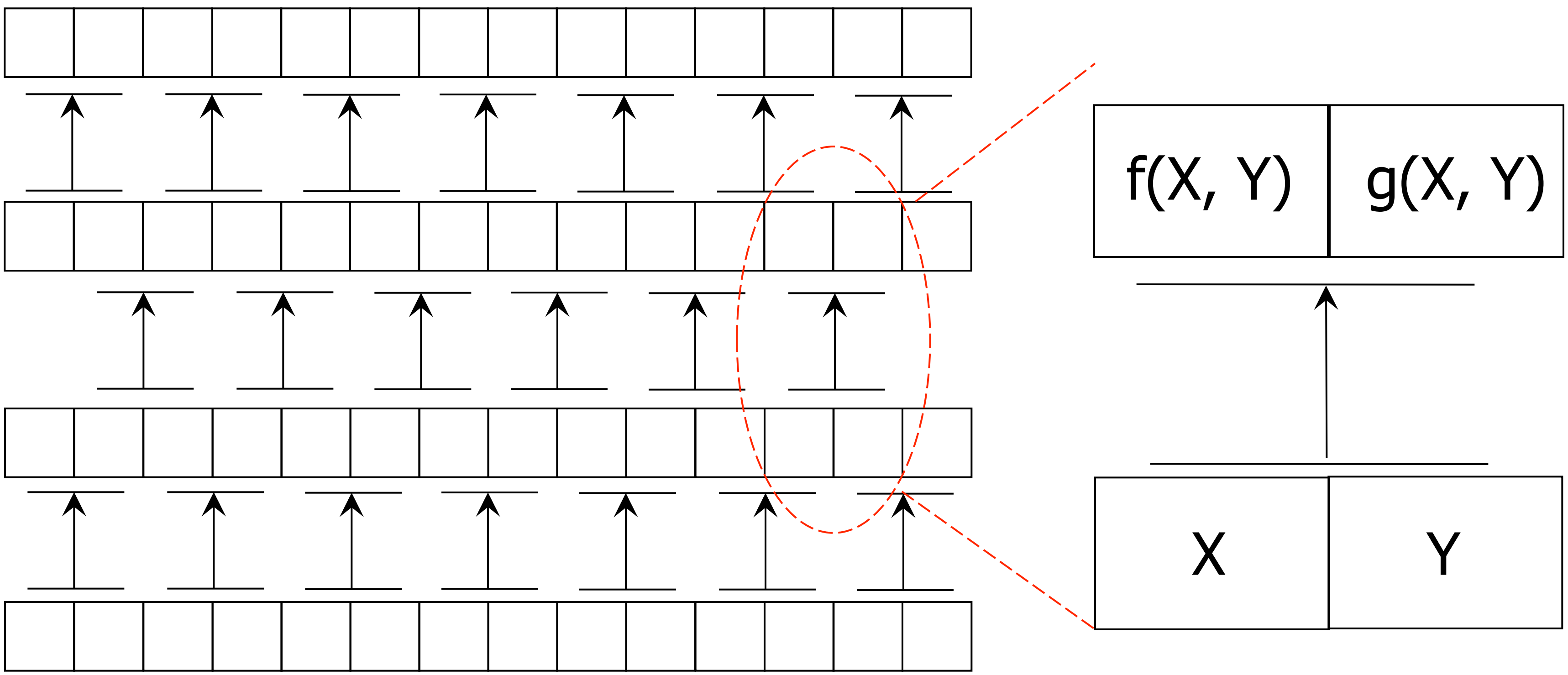
Paul Rothemund, Nick Papadakis, Erik Winfree, PLoS Biology 2: e424 (2004)



Theoretical Results

- Efficiently assembling basic shapes with precisely controlled size and pattern.
  - Constructing N X N squares with O(log n/log log n) tiles. [Adleman, Cheng, Goel, Huang, ’ 01]
  - Perform universal computation by simulating BCA.
    [Winfree ’ 99]
  - Assemble arbitrary shape by O(Kolmogorov complexity) tiles with scaling. [Soloveichik, Winfree ’ 04]

Block Cellular Automata



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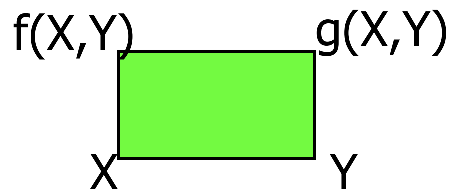
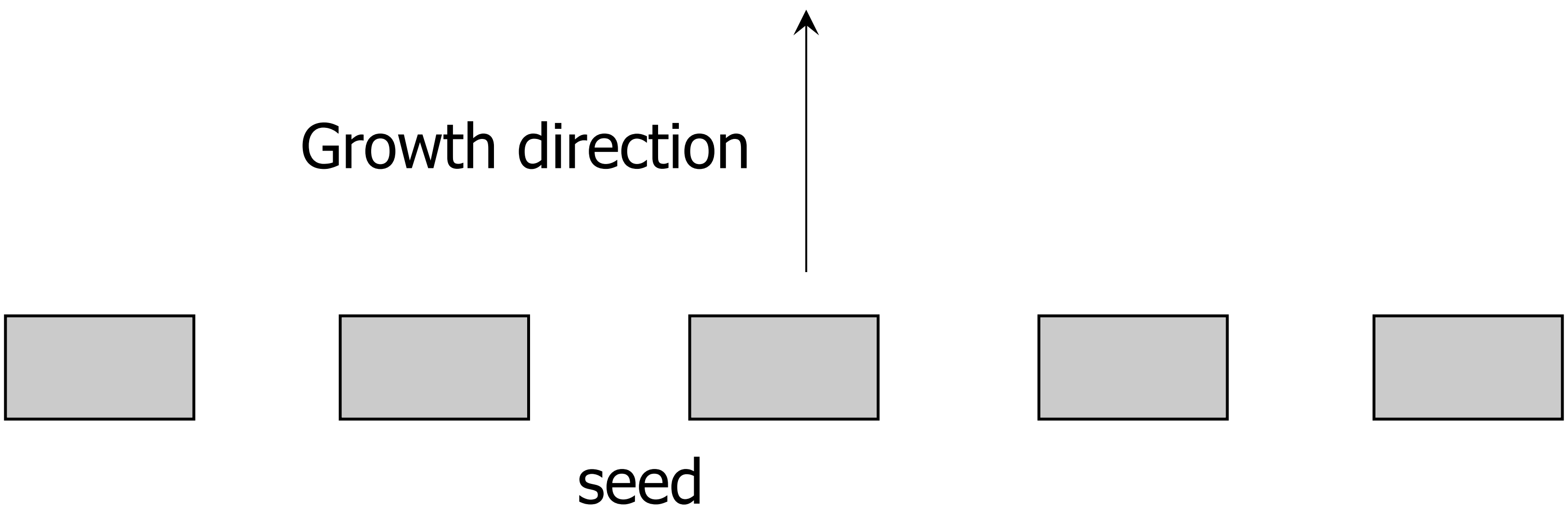
Simulating BCA

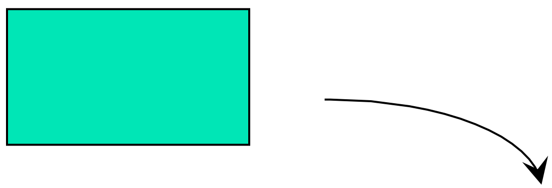
T=2

A series of tiles with format:

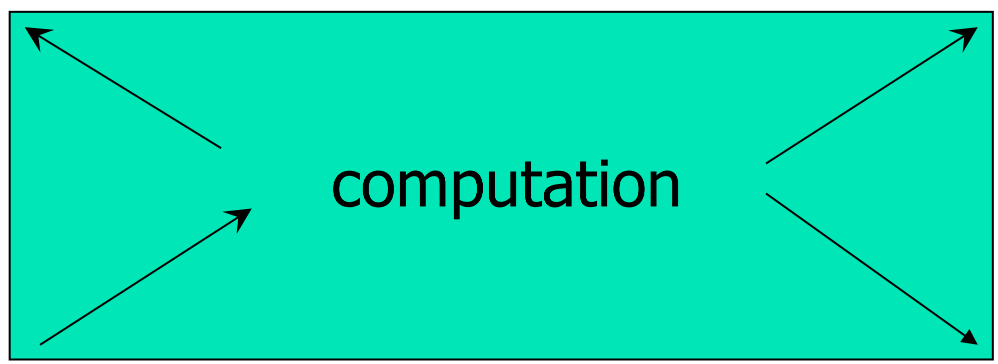




Assemble Arbitrary Shapes



Replace each tile by a block.
Size of block = O(computation time)



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Tile System Design

- Library of primitives to use in designing nano-scale structures

  [Adleman, Cheng, Goel, Huang, ’01]

- Automate the design process

  [Adleman, Cheng, Goel, Huang, Kempe, Moisset de espanes, Rothemund ’01]

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Kinetic System Model

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kinetic Tile Assembly Model:

[Winfree, 1998]

A tile can attach

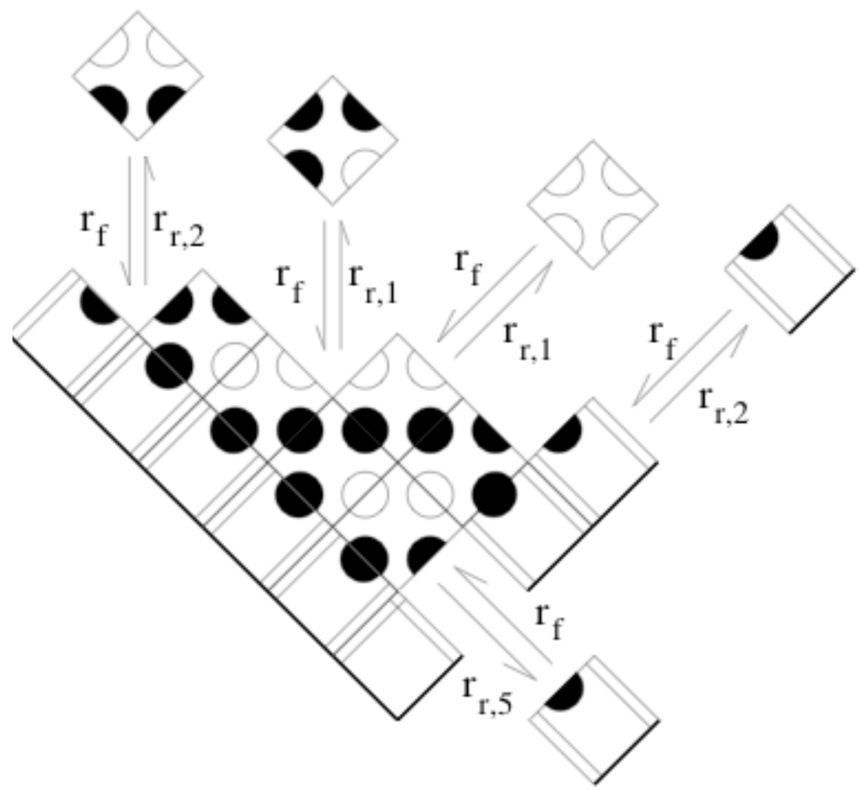
at any location.

The rate of attachment

r_f = constant.

The rate of detachment

$r_{r,b} = c\ e^{-bG}$



Kinetic model => Abstract model

- We set the temperature and concentration to

$$r_{r,T+1} \ll r_{r,T} < r_f \ll r_{r,T-1}$$

  - If a tile attaches with strength < T-1, it is likely to fall off very fast.
  - If a tile is held by strength at least T+1, it is unlikely to fall off

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